Title: **Random Polynomials Having Few or No Real Zeros**

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Abstract:

Consider a polynomial of large degree n whose coefficients are independent, identically distributed, nondegenerate random variables having zero mean and finite moments of all orders. We show that such a polynomial has exactly k real zeros with probability $n^{-b+o(1)}$ as $n \to \infty$ through integers of the same parity as the fixed integer $k \geq 0$. In particular, the probability that a random polynomial of large even degree n has no real zeros is $n^{-b+o(1)}$. The finite, positive constant b is characterized via the centered, stationary Gaussian process of correlation function $\operatorname{sech}(t/2)$. The value of b depends neither on k nor upon the specific law of the coefficients. Under an extra smoothness assumption about the law of the coefficients, with probability $n^{-b+o(1)}$ one may specify also the approximate locations of the k zeros on the real line. The constant b is replaced by $b/2$ in case the i.i.d. coefficients have a nonzero mean.