Title:

Cover Times for Brownian Motion and Random Walks in two Dimensions

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Abstract:

Let $(x,)$ denote the first hitting time of the disc of radius centered at x for Brownian motion on the two dimensional torus T^2. We prove that $\sup_{x \in T^2} (x,)/|\log |^2 \to 2/\pi$ as 0. The same applies to Brownian motion on any smooth, compact connected, two-dimensional, Riemannian manifold with unit area and no boundary. As a consequence, we prove a conjecture, due to Aldous (1989), that the number of steps it takes a simple random walk to cover all points of the lattice torus ${}^{2}_{n}$ is asymptotic to $4n^2(\log n)^2/\pi$. Determining these asymptotics is an essential step toward analyzing the fractal structure of the set of uncovered sites before coverage is complete; so far, this structure was only studied non-rigorously in the physics literature. We also establish a conjecture, due to Kesten and Révész, that describes the asymptotics for the number of steps needed by simple random walk in ^{2} to cover the disc of radius n.