Title:

**The Minimax Distortion Redundancy in Noisy Source Coding**

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Abstract:

Consider the problem of finite-rate filtering of a discrete memoryless process $\{X_i\}_{i\geq 1}$ based on its noisy observation sequence $\{Z_i\}_{i\geq 1}$, which is the output of a Discrete Memoryless Channel (DMC) whose input is $\{X_i\}_{i\geq 1}$. When the distribution of the pairs $(X_i, Z_i)$, $P_{X,Z}$, is known, and for a given distortion measure, the solution to this problem is well known to be given by classical rate-distortion theory upon the introduction of a modified distortion measure. In this work we address the case where $P_{X,Z}$, rather than being completely specified, is only known to belong to some set $\Lambda$. For a fixed encoding rate $R$ we look at the worst case, over all $\theta \in \Lambda$, of the difference between the expected distortion of a given scheme which is not allowed to depend on the active source $\theta \in \Lambda$ and the value of the distortion-rate function at $R$ corresponding to the noisy source $\theta$. We study the minimum attainable value achievable by any scheme operating at rate $R$ for this worst-case quantity, denoted by $D(\Lambda, R)$. Linking between this problem and that of source coding under several distortion measures, we prove a coding theorem for the latter problem and apply it to characterize $D(\Lambda, R)$ for the case where all members of $\Lambda$ share the same noisy marginal. For the case of a general $\Lambda$, we obtain a single-letter characterization of $D(\Lambda, R)$ for the finite-alphabet case. This gives, in particular, a necessary and sufficient condition on the set $\Lambda$ for the existence of a coding scheme which is universally optimal for all members of $\Lambda$ and characterizes the approximation-estimation tradeoff for statistical modelling of noisy source coding problems. Finally, we obtain $D(\Lambda, R)$ in closed form for cases where $\Lambda$ consists of distributions on the (channel) input-output pair of a Bernoulli source corrupted by a Binary Symmetric Channel (BSC). In particular, for the case where $\Lambda$ consists of two sources: the all-zero source corrupted by a BSC with crossover probability $r$ and the Bernoulli($r$) source with a noise-free channel; we find that universality becomes increasingly hard with increasing rate.