

Title:

**On the Maximum Correlation Coefficient**

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Abstract:

For an arbitrary random vector  $(X, Y)$  and an independent random variable  $Z$  it is shown that the maximum correlation coefficient between  $X$  and  $Y + \lambda Z$  as a function of  $\lambda$  is lower semi-continuous everywhere and continuous at zero where it attains its maximum. If, moreover,  $Z$  is in the class of self-decomposable random variables, then the maximal correlation coefficient is right continuous, non-increasing for  $\lambda \geq 0$  and left continuous, non-decreasing for  $\lambda \leq 0$ . Independent random variables  $X$  and  $Z$  are Gaussian if and only if the maximum correlation coefficient between  $X$  and  $X + \lambda Z$  equals the linear correlation between them. The maximum correlation coefficient between the sum of  $n$  arbitrary independent identically distributed random variables and the sum of the first  $m < n$  of these, equals  $\sqrt{m/n}$  (previously proved only for random variables with finite second moments, where it amounts also to the linear correlation). Examples provided reveal counterintuitive behavior of the maximum correlation coefficient for more general  $Z$  and in the limit  $\lambda \rightarrow \infty$ .