Title:
When Does ISOMAP Recover the Natural Parameterization of Families of Articulated Images?

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Abstract:

In science and engineering, there is a need to observe high dimensional data and "learn" a potential underlying nonlinear parametrization for the data. Recently, the ISOMAP procedure was proposed as a new way to recover hidden parametrizations of high-dimensional data. We consider a specific kind of data – families of images generated by the articulation of an object – in an idealization where the images are functions on the continuum plane. Using the ambient L^2-distance as a metric between articulated images makes the articulation family a nonlinear manifold. We introduce continuum ISOMAP, an analog of the ISOMAP procedure where we obtain geodesic distance between points of the articulation manifold and attempt to realize that distance as a Euclidean metric on a Euclidean space. We study the question of when Continuum ISOMAP can truly recover the underlying parametrization of a family of images. We show that for images with edges, this idealization suffers from various infinities, but a natural renormalization of the notion of geodesic distance is well-defined. We exhibit a set of interesting image manifolds where the geodesic distance on the manifold is exactly proportional to the Euclidean distance in parameter space, and therefore, Continuum ISOMAP works perfectly to recover the natural parametrization up to a rigid motion. Examples of such successes include: translations of a disk, rotations of a closed figure, articulations of a horizon, independent non-occluding motions of 'fingers' of a cartoon 'hand', and gestures of a cartoon 'face', with articulated features.The theoretical predictions of the Continuum ISOMAP model are borne out by empirical experiments with published ISOMAP code. However, we demonstrate that in the case where several components of the image articulate independently and occlusion is possible, recovery of the original parameter space may be precluded by the characteristics of the data manifold. This methodology suggests new modifications to the existing procedures for recovering underlying image parametrization.