Title:

On the Largest Eigenvalue of Wishart Matrices wth Identity Covariance when n, p and $p/n \to \infty$

Author(s):

Noureddine El Karoui

Technical Report number (Dept. of Statistics, Stanford Univ.):

2003-25

Date:

September 2003

Abstract:

Let X be a $n \times p$ matrix and l_1 the largest eigenvalue of the covariance matrix X^*X. The "null case" where $X_{i,j} \sim \mathcal{N}(0,1)$ is of particular interest for principal component analysis. For this model, when $n, p \to \infty$ and $n/p \to \gamma \in \mathbf{R}_+^*$, it was shown in Johnstone (2001) that l_1, properly centered and scaled, converges to the Tracy-Widom law. We show that with the same centering and scaling, the result is true even when p/n or $n/p \to \infty$, therefore extending the previous result to $\gamma \in \overline{\mathbf{R}}_+$. The derivation uses ideas and techniques quite similar to the ones presented in Johnstone (2001). Following Soshnikov (2002), we also show that the same is true for the joint distribution of the k largest eigenvalues, where k is a fixed integer.

Numerical experiments illustrate the fact that the Tracy-Widom approximation is reasonable even when one of the dimension is small.