Title:
Multidimensional variation for quasi-Monte Carlo

Author(s):
Art B. Owen

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abstract

Dedicated to Professor Fang Kai-Tai in honor of his 65th birthday

This paper collects together some properties of multidimensional definitions of the total variation of a real valued function. The subject has been studied for a long time. Many of the results presented here date back at least to the early 1900s.

The main reason for revisiting this topic is that there has been much recent work in theory and applications of Quasi-Monte Carlo (QMC) sampling. For an account of quasi-Monte Carlo integration see Fang and Wang (1994) and Niederreiter (1992). QMC is especially competitive for multidimensional integrands with bounded variation in the sense of Hardy and Krause (BVHK). For such integrands, over d dimensional domains, one sees QMC errors that are $O(n^{-1}(\log n)^d)$ when using n function evaluations. When $d = 1$, competing methods are usually preferred to QMC. For even modestly large d, Monte Carlo and quasi-Monte Carlo sampling become the methods of choice.

When the integrand is in BVHK, then QMC has superior asymptotic behavior, compared to Monte Carlo sampling. Therefore we may like to know when a specific function is in BVHK.