Title:
Spectral Measure of Large Random Hankel, Markov and Toeplitz Matrices

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Abstract:

We study the limiting spectral measure of large symmetric random matrices of linear algebraic structure.

For Hankel and Toeplitz matrices generated by i.i.d. random variables $\{X_k\}$ of unit variance, and for symmetric Markov matrices generated by i.i.d. random variables $\{X_{i,j}\}_{j>i}$ of zero mean and unit variance, scaling the eigenvalues by $\sqrt{n}$ we prove the almost sure, weak convergence of the spectral measures to universal, non-random, symmetric distributions γ_H, γ_M, and γ_T of unbounded support. The moments of γ_H and γ_T are the sum of volumes of solids related to Eulerian numbers, whereas γ_M has a bounded smooth density given by the free convolution of the semi-circle and normal densities.

For symmetric Markov matrices generated by i.i.d. random variables $\{X_{i,j}\}_{j>i}$ of mean m and finite variance, scaling the eigenvalues by n we prove the almost sure, weak convergence of the spectral measures to the atomic measure at $-m$. If $m = 0$, and the fourth moment is finite, we prove that the spectral norm of such matrices scaled by $\sqrt{2n \log n}$ converges almost surely to one.