Title:
Extensions of Compressed Sensing

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Abstract:

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Extensions of Compressed Sensing

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Abstract

We study the notion of Compressed Sensing (CS) as put forward in [?] and related work [?, ?, ?]. The basic idea behind CS is that a signal or image, unknown but supposed to be compressible by a known transform, (eg. wavelet or Fourier), can be subjected to fewer measurements than the nominal number of pixels, and yet be accurately reconstructed. The samples are nonadaptive and measure 'random' linear combinations of the transform coefficients. Approximate reconstruction is obtained by solving for the transform coefficients consistent with measured data and having the smallest possible ℓ^1 norm.

We perform a series of numerical experiments which validate in general terms the basic idea proposed in [?, ?, ?], in the favorable case where the transform coefficients are sparse in the strong sense that the vast majority are zero. We then consider a range of less-favorable cases, in which the object has all coefficients nonzero, but the coefficients obey an ℓ^p bound, for some $p \in (0, 1]$. These experiments show that the basic inequalities behind the CS method seem to involve reasonable constants.

We next consider synthetic examples modelling problems in spectroscopy and image processing, and note that reconstructions from CS are often visually "noisy" . We post-process using translation-invariant de-noising, and find the visual appearance considerably improved.

We also consider a multiscale deployment of compressed sensing, in which various scales are segregated and CS applied separately to each; this gives much better quality reconstructions than a literal deployment of the CS methodology.

We also report that several workable families of 'random' linear combinations all behave equivalently, including random spherical, random signs, partial Fourier and partial Hadamard.

These results show that, when appropriately deployed in a favorable setting, the CS framework is able to save significantly over traditional sampling, and there are many useful extensions of the basic idea.