**Finite Sample Nonparametric Inference and Large Sample Efficiency**

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Abstract:

Given a sample $X_1, \ldots, X_n$ from a distribution $F$, the problem of constructing nonparametric confidence intervals for the mean $\mu(F)$ is considered. Unlike bootstrap procedures or those based on normal approximations, we insist on any procedure being truly nonparametric in the sense that the probability that the confidence interval contains $\mu(F)$ based on a sample of size $n$ from $F$ be at least $1-\alpha$ for all $F$ and all $n$. Bahadur and Savage (1956) proved it is impossible to find an effective confidence interval for $\mu(F)$ without some restrictions. Thus, we assume that $F$ is supported on a compact set, which we take to be $[0,1]$. In this setting, an asymptotic efficiency result is obtained that gives a lower bound on the size of any conservative interval. We then provide a construction of an interval that meets our finite sample requirement on level, yet has an asymptotic efficiency property. Thus, the price to be paid for using fully nonparametric procedures when considering the tradeoff between exact inference statements and asymptotic efficiency is negligible. Much of what is accomplished for the mean generalizes to other settings as well.