

## **Geometry and Probability in Three Dimensions**

Authors: **Louis J. Billera, Kenneth S. Brown, and Persi Diaconis**

Technical Report number (Dept. of Statistics, Stanford University): **1998-13**

Date: **August 1998**

Abstract:

This paper explains some modern geometry and probability in the course of solving a random walk problem. Consider  $n$  planes through the origin in three dimensional Euclidean space. Assume, for simplicity, that they are in general position. They then divide space into  $n(n - 1) + 2$  regions. We study a random walk on these regions. Suppose the walk is in region  $C$ . Pick a pair of the planes at random. These determine a line through the origin. Pick one of the two halves of the line with equal probability. The walk now moves to the region adjacent to the chosen half-line which is closest to  $C$ . We determine the long-term stationary distribution: All regions of  $i$  sides have stationary probability proportional to  $i - 2$ . We further show that the walk is close to its stationary distribution after two steps.