Ridgelets: Theory and Applications

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Abstract:

Single hidden-layer feedforward neural networks have been proposed as an approach to bypass the curse of dimensionality and are now becoming widely applied to approximation or prediction in applied sciences. In that approach, one approximates a multivariate target function by a sum of ridge functions; this is similar to projection pursuit in the literature of statistics. This approach poses new and challenging questions both at a practical and theoretical level, ranging from the construction of neural networks to their effciency and capability. The topic of this thesis is to show that *ridgelets*, a new set of functions, provide an elegant tool to answer some of these fundamental questions.

In the first part of the thesis, we introduce a special admissibility condition for neural activation functions. Using an admissible neuron, we develop two linear transforms, namely the continuous and discrete ridgelet transforms. Both transforms represent quite general functions f as a superposition of ridge functions in a stable and concrete way. A frame of "nearly orthogonal" ridgelets underlies the discrete transform.

In the second part, we show how to use the ridgelet transform to derive new approximation bounds. That is, we introduce a new family of smoothness classes and show how they model "real-life" signals by exhibiting some specific sorts of high-dimensional spatial inhomogeneities. Roughly speaking, finite linear combinations of ridgelets are optimal for approximating functions from these new classes. In addition, we use the ridgelet transform to study the limitations of neural netrowks. As a surprising and remarkable example, we discuss the case of approximating radial functions.

Finally, it is explained in the conclusion why these new ridgelet expansions offer decisive improvements over traditional neural networks.