Tight Frames of k-Plane Ridgelets and the Problem of Representing Objects Which Are Smooth Away from d-Dimensional Singularities in $\mathbf{R}^n$

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Abstract:

For each pair (n, k) with $1 \leq k < n$, we construct a tight frame $(\rho_\lambda : \lambda \in \Lambda)$ for $L^2(\mathbf{R}^n)$, which we call a frame of *k-plane ridgelets*. The intent is to efficiently represent functions which are smooth away from singularities along k-planes in $\mathbf{R}^n$. We also develop tools to help decide whether in fact k-plane ridgelets provide the desired efficient representation.

We first construct a wavelet-like tight frame on the *X-ray bundle* $\mathcal{X}_{n,k}$ — the fiber bundle having the Grassman manifold $G_{n,k}$ of k-planes in $\mathbf{R}^n$ for base space, and for fibers the orthocomplements of those planes. This wavelet-like tight frame is the pushout to $\mathcal{X}_{n,k}$, via the smooth local coordinates of $G_{n,k}$, of an orthonormal basis of tensor Meyer wavelets on Euclidean space $\mathbf{R}^{k(n-k)} \times \mathbf{R}^{n-k}$. We then use the *X-ray isometry* [Solmon, 1976] to map this tight frame isometrically to a tight frame for $L^2(\mathbf{R}^n)$ – the k-plane ridgelets.

This construction makes analysis of a function $f \in L^2(\mathbf{R}^n)$ by k-plane ridgelets identical to the analysis of the k-plane X-ray transform of f by an appropriate wavelet-like system for $\mathcal{X}_{n,k}$. As wavelets are typically effective at representing point singularities, it may be expected that these new systems will be effective at representing objects whose k-plane X-ray transform has a point singularity. Objects with discontinuities across hyperplanes are of this form, for $k = n - 1$.