Title: **Theory and Applications of Decoupling**

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Abstract:

In this paper we provide a survey of the theory of decoupling, emphasizing its wide range of applications. Decoupling was born out of a need to extend the known martingale inequalities for real and Hilbert-space valued random variables to variables taking values in more general spaces like Banach spaces. Among its many uses is that it enables one to symmetrize highly dependent random variables. In fact, some of the basic results in decoupling theory were motivated by symmetrization techniques for U-processes which consist of U-statistics indexed by a class of functions. Other applications include sequential analysis, martingale theory, stochastic integration and weak convergence. We consider three types of decoupling. The first type, which we call complete decoupling, completely replaces the summands in a sum of dependent random variables by independent ones with the same marginal distributions. The second type of decoupling is decoupling of tangent sequences. In this approach, two sequences adapted to the same filtration and having the same conditional distributions (given the past) are compared. The effectiveness of this approach is realized through the use of a sequence with a more tractable dependence structure than the original one. The third type of decoupling is called "total decoupling of stopping times". In this approach, problems involving stopping times are handled by establishing inequalities that replace the original process by an independent copy which is therefore independent of the stopping time.