STANFORD UNIVERSITY

DEPARTMENT OF STATISTICS

DEPARTMENTAL SEMINAR

4:15 p.m., Tuesday, January 30, 2001
Sequoia Hall Rm. 200
(Cookies at 3:45 in 1st Floor Lounge)

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Random fields on surfaces and volumes without stationarity: Applications to brain imaging

An important tool in signal detection in brain imaging is the distribution of the maximum of a random field on some Euclidean space, which is taken to be the parameter space of the random field. One approach that has proved successful in approximating the tail of this distribution is the so-called EC or (expected) Euler characteristic approach. This approach uses techniques from integral geometry to approximate the distribution of the maximum of the random field and provides an approximation of the tail in terms of certain integral invariants of S, known in integral geometry as the intrinsic volumes of S.

A fundamental assumption of this approach was that the random field was isotropic, i.e. invariant under the group of Euclidean rigid motions. However, when studying random fields on manifolds such as the cortical surface, there is no notion of isotropy because of the irregularity of the manifold. Further, the distribution of the maximum of the random field is invariant under diffeomorphisms of the parameter space, so that our inference should not be based on how we visualize the data, i.e., flattening the cortical surface to view data in the sulci of the cortical surface should not affect the inference about the presence of a signal or not.

These ideas point to the fact that the relevant geometry in the study of such random fields is one that is intrinsic to the random field itself. In other words, the relevant geometry is derived from the Riemannian structure that the random field induces on the manifold and not (necessarily) from the space in which we visualize the data. In the case of an isotropic field on a Euclidean space, the assumption of isotropy restricts the geometry to be the geometry of the Euclidean space (modulo a constant).

We describe the EC approach and how it can be extended smoothly to random fields on manifolds as well as how to estimate the relevant geometric quantities that appear in the EC approximation. We illustrate the method on some fMRI (functional magnetic resonance imaging) data, restricted to the cortical surface.

If time permits, we will describe a version of the classical Kinematic Fundamental Formula from integral geometry for smooth vector-valued Gaussian random fields. This has applications for the EC approach in a non-Gaussian framework, i.e. for random fields built from i.i.d. Gaussian fields.