STANFORD UNIVERSITY

DEPARTMENT OF STATISTICS

DEPARTMENT SEMINAR

4:15 p.m., Wednesday, July 24, 2002
Sequoia Hall Room 200
(Cookies at 3:45 in 1st Floor Lounge)

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Optimal designs for estimating the path of a stochastic process

A second-order random process $Y(t)$, with $E(Y(t)) \equiv 0$, is sampled at a finite number of design points $t_1, t_2, ..., t_n$. On the basis of these observations, one wants to estimate the values of the process at unsampled points using the best linear unbiased estimator (BLUE). The performance of the estimator is measured by a weighted integrated mean square error. The goal is to find $t_1, t_2, ..., t_n$, such that this integrated mean square error (IMSE) is minimized for a fixed n.

This optimization problem depends on the stochastic process only through its covariance structure. For processes with a product type covariance structure, i.e., for $Cov(Y(s), Y(t)) = u(s)v(t)$, $s < t$, a set of necessary and sufficient conditions for a design to be exactly optimal will be presented. Starting from the set of exact optimality conditions for a fixed n, an asymptotic result yielding the density whose percentile points furnish a set of asymptotically optimal design points (in some suitable sense) will be described. Results on the problem when one tries to estimate the integral of $Y(t)$ instead of the path will be discussed briefly. The integral estimation problem is related to certain regression design problems with correlated errors discussed by Sacks and Ylvisaker in a series of papers (1966,1968,1970).

For a more general covariance structure, satisfying Sacks-Ylvisaker regularity conditions, it is noted that a much simpler estimator is asymptotically equivalent to the BLUE. This leads to an intuitively appealing argument in establishing the asymptotic behaviour of the BLUE and also in deriving an analytical expression for the asymptotically optimal design density.