

STANFORD UNIVERSITY
DEPARTMENT OF STATISTICS
DEPARTMENTAL SEMINAR

4:15 p.m., Tuesday, March 18, 2003
Sequoia Hall Room 200
(Cookies at 3:45 in 1st Floor Lounge)

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Fixed-Domain Asymptotics for Gaussian Random Fields with Matern-Type Covariance

In the modeling of computer experiments, it has become rather common practice to approximate the deterministic response as a realization of a stochastic process. In this regard, Jerry Sacks, et. al. (*Statist. Sci.*, 1989) proposed modeling using a Gaussian random field $X(x)$, $x \in [0, 1]^d$, with a multiplicative covariance function

$$\text{Cov}(X(x), X(y)) = \sigma^2 \prod_{t=1}^d \exp(-\theta_t |x_t - y_t|^\gamma), \quad x = (x_1, \dots, x_d)', y = (y_1, \dots, y_d)',$$

where $\gamma \in (0, 2]$, $\theta_1, \dots, \theta_d$ and σ^2 are strictly positive parameters.

Michael Stein (*Statist. Sci.*, 1989) observed that the above Gaussian model may have some undesirable properties. In particular for $\gamma \in (0, 2)$, the Gaussian random field with this covariance function will not be mean square differentiable. However for the case $\gamma = 2$, it is infinitely mean square differentiable. Not allowing for processes that are differentiable but not infinitely differentiable may be unnecessarily restrictive. Stein further suggested using a Gaussian random field model, $X(x)$, $x \in [0, 1]^d$, with the multiplicative Matérn-type covariance function

$$\text{Cov}(X(x), X(y)) = \prod_{t=1}^d \pi^{1/2} \phi 2^{\alpha-1} \Gamma(\alpha + 1/2) \theta_t^{2\alpha} (\theta_t |x_t - y_t|)^\alpha K_\alpha(\theta_t |x_t - y_t|)$$

where $\alpha, \phi, \theta_1, \dots, \theta_d$ are positive constants and K_α is the modified Bessel function of the second kind. The interesting point is that X will be m times mean square differentiable if and only if $\alpha > m$.

In this talk, we shall focus on some fixed-domain asymptotic results for Gaussian random fields with $\alpha = 3/2$ multiplicative Matérn-type covariance functions.

For a PDF version of the above abstract, please see the attached file.