

STANFORD UNIVERSITY
DEPARTMENT OF STATISTICS
DEPARTMENTAL SEMINAR

4:15 p.m., Wednesday, February 1, 2006
Sequoia Hall Room 200
(Cookies at 3:45 in 1st Floor Lounge)

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On Conjugacy and Self-Consistency in Bayesian Inference

The notion of self-consistency of inferential procedures has arisen in a number of statistical contexts. In a survival analysis setting, Efron (Proc. Berkeley Symp., 1967) defined self consistency in terms of a natural recursive relationship that nonparametric estimators might satisfy, and showed that the Kaplan-Meier estimator was the unique self-consistent estimator of the survival function on the interval containing all deaths and censoring times. Tsai and Crowley (Ann. Stat., 1985) identified self consistent estimators as the unique fixed points of nonparametric EM algorithms. In this talk, self consistency is defined in the context of Bayes estimation, relative to squared error loss, of a parameter θ of an exponential family of distributions. We define a prior distribution (or Bayes estimator) as self consistent if the equation $E(\theta|T = E(\theta)) = E(\theta)$ is satisfied, where T is a sufficient and unbiased estimator of θ (or the UMVUE of θ under mild additional assumptions). This equation simply states that, if your experimental outcome agrees with your prior opinion about the parameter, then the experiment should not change your opinion about θ . While this condition seems natural and compelling, many prior distributions, including both objective and proper priors, do not enjoy this property. Some characterization results for families of self-consistent priors are obtained. The concept of conjugacy will be broadened by relaxing the exponentiality condition imposed on the prior model by Diaconis and Ylvisaker (Ann. Stat., 1979) while retaining their essential condition of a linear posterior mean. I'll conclude by specifying when Bayes estimators relative to priors belonging to a broad conjugate and self-consistent family outperform classical procedures (in the sense of Samaniego and Reneau (JASA, 1994)).