

STANFORD UNIVERSITY
DEPARTMENT OF STATISTICS
DEPARTMENTAL SEMINAR

4:15 p.m., Tuesday, June 27, 2006
Sequoia Hall Room 200
(Cookies at 3:45 in 1st Floor Lounge)

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Brownian Motion and the Unitary Group

A theorem of Borel (1906) implies that if Γ is chosen from the orthogonal group O_n distributed according to Haar measure and γ_n is any of its entries, then $\sqrt{n}\gamma_n$ converges in distribution to a standard normal random variable as $n \rightarrow \infty$. In joint work with Diaconis and Newman, this result was greatly extended by showing that if A is an $n \times n$ real non-random matrix satisfying $\text{Tr}(AA^t) = n$, then the trace of the matrix $A\Gamma$ converges in distribution to a standard normal random variable as $n \rightarrow \infty$. (The matrices Γ and A depend on n , but we have suppressed this dependence in the notation.) This theorem is then applied to show that appropriately normalized entries from Γ can replace i.i.d. standard normal random variables in the usual construction of Brownian motion. There were also attempts to provide corresponding theorems for the unitary group and complex Brownian motion but here the authors were only partially successful. In this talk, we will provide completely parallel results for the unitary group. Along the way, we will give sufficient conditions for convergence in distribution on the space $(D \times D, \mathcal{E})$ where D is the Skorohod space of right-continuous functions on $[0,1]$ with left limits and \mathcal{E} is the σ -algebra of Borel sets of $D \times D$. This extends results of Billingsley from his book *Convergence of Probability Measures*. We will also provide the analogue for the unitary group of a result of Charles Stein concerning the expectation of certain products of elements from a random orthogonal matrix.